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Electron–photon–phonon interactions in polar semiconductors under free-electron laser irradiations

W Xu†

Department of Engineering Physics, Institute for Superconducting and Electronic Materials, University of Wollongong, Wollongong, NSW 2522, Australia

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Abstract. In this paper, I present a detailed theoretical study of electron–photon–phonon interactions in three-dimensional electron gases (3DEGs) subjected to linearly polarized intense electromagnetic (EM) radiations. Applying the solution of the time-dependent Schrödinger equation, in which the effect of the EM radiation field is included exactly, to time-dependent perturbation theory, I have developed a novel approach for determining the probabilities of steady-state electronic transitions induced by electron–photon–phonon interactions in a 3DEG system. For the case of polar semiconductors, I have discussed the influence of the linearly polarized intense laser radiation on the electronic scattering rate for electron interactions with the radiation field and with the LO phonons. These results are pertinent to the application of recently developed terahertz or far-infrared laser sources such as free-electron lasers.

1. Introduction

In recent years, there has been a rapid expansion worldwide in developing coherent, high-power, long-wavelength and tunable electromagnetic (EM) radiation sources such as free-electron lasers (FELs). The FELs are generated via passing an intense beam of relativistic electrons through periodic magnetic fields and can provide linearly polarized laser radiations. The current generation of the FELs, in operation in, e.g., UCSB [1–3] and FELIX [4–6], has already been able to provide a tunable source of intense laser radiations in the terahertz (THz) or far-infrared (FIR) bandwidth. Since 1995 [1, 5], the THz FEL radiations have been successfully applied in scientific research into non-linear transport and optical properties in different semiconductor devices.

The significant impact of intense THz laser radiations provided by FELs on semiconductor physics and electronics can be understood by considering the fact that when an electron gas, realized in, e.g., a semiconductor, is subjected to an intense THz EM field, the electron kinetic energy, the Fermi energy, the phonon energy, the plasmon energy etc in the system are comparable to the energy of the THz photons and to that of the radiation field. As a result, the applied THz EM field can couple strongly to the electronic system. In this situation, the electrons can interact with the radiation field via emission and absorption of THz photons. Furthermore, for a THz-driven electron gas in a semiconductor device, the rate of scattering induced by electron interactions with impurities and phonons can be of the same order as the THz photon frequency [7]. This implies that the THz radiations can significantly modify the processes of momentum and energy relaxation for excited electrons

† E-mail: wxu@wumpus.its.uow.edu.au.

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in the device system. Very recently, some important and interesting THz phenomena, such as resonant absorption of THz radiations [1], the THz-radiation-enhanced hot-electron effect [2], THz-photon-induced impact ionization [3], the LO-phonon bottleneck effect [4], THz-photon-assisted resonant tunnelling [5] and FIR cyclotron resonance [6] have been observed experimentally in different semiconductor structures using FEL radiations. When a polar semiconductor such as GaAs is subjected to an intense THz EM field, optical excitation of electrons can occur. Due to the presence of the strong phonon oscillation modes and to the comparability of the frequencies of the phonons and photons, the electron–phonon interaction is the principal channel for relaxation of the excited electrons in a polar semiconductor. The results obtained from experimental measurements in [1, 2, 4, 6] have indicated that phonons play an important role in non-linear transport and the optical properties observed for THz-driven electron gases.

The brief review given above shows that in order to achieve a better understanding of the experimental results reported very recently, more theoretical work on electron–photon–phonon interactions in an electron gas system, in particular for GaAs-based systems subjected to intense THz FEL radiations, is required. In the past, most theoretical approaches dealing with electron–photon–phonon interactions [8, 9] were based on separate Hamiltonians which describe respectively the electron–phonon and electron–photon interactions. It can be demonstrated, as can be seen in this paper, that this approach can only be used for the case of high-frequency and/or low-intensity EM radiations. In this paper, I develop a rather simple theoretical approach by means of which one can easily study the electron–photon–phonon interactions over a wide radiation intensity and frequency range. This approach has gone beyond the conventional treatment for electron–photon–phonon interactions in semiconductor systems in the presence of radiation fields. In section 2, I examine the properties of the unitary operator for an ideal 3DEG in the presence of the EM radiations, using time-dependent perturbation theory. With the unitary operator obtained for the system in the presence of the scattering potential, the first-order contribution to the probability of steady-state electronic transitions, induced by electron–photon–phonon coupling, is obtained in section 3. In section 4, I discuss the influence of the intensity and frequency of the FEL radiation on the rate of electronic scattering caused by interactions of electrons with photons and with LO phonons in GaAs-based 3DEG systems. The conclusions obtained from the present study are summarized in section 5.

2. The unitary operator

In this paper, I consider an electronic system which can be described by a single-electron Hamiltonian: $H(t) = H_0(t) + V(t)$, where $V(t)$ can be treated as a perturbation. For an ideal 3DEG system subjected to an EM radiation field polarized along the x-axis, the Hamiltonian for a non-interacting electron can be written as

$$H_0(t) = \frac{[p_x - eA(t)]^2 + p_y^2 + p_z^2}{2m^*}. \tag{1a}$$

Here: (i) a parabolic-conduction-band structure has been included; (ii) $p_x = -\mathrm{i}\hbar\,\partial/\partial x$ is the momentum operator; (iii) $\boldsymbol{A}(t)$ is the vector potential induced by the EM radiation; (iv) m^* is the effective electron mass. Furthermore, I have used the Coulomb gauge [10] to describe the EM radiation field. After using the dipole approximation for the radiation field and taking $A(t) = A_0 \sin(\omega t)$, with ω being the frequency of the EM field, the solution of the time-dependent Schrödinger equation, $\mathrm{i}\hbar\,\partial\psi(\boldsymbol{R}, t)/\partial t = H_0(t)\psi(\boldsymbol{R}, t)$, is obtained as

$$|\boldsymbol{K}; t\rangle = \psi_{\boldsymbol{K}}(\boldsymbol{R}, t) = |\boldsymbol{K}; 0\rangle \mathrm{e}^{-\mathrm{i}[E(\boldsymbol{K})+2\gamma\hbar\omega]t/\hbar} \mathrm{e}^{\mathrm{i}r_0 k_x[1-\cos(\omega t)]} \mathrm{e}^{\mathrm{i}\gamma\sin(2\omega t)} \tag{1b}$$

where $|\boldsymbol{K};0\rangle = |\boldsymbol{K}\rangle = \mathrm{e}^{\mathrm{i}\boldsymbol{K}\cdot\boldsymbol{R}}$ is the electron wavefunction at time $t = 0$, $\boldsymbol{R} = (x, y, z)$, $\boldsymbol{K} = (k_x, k_y, k_z)$ is the electron wavevector, $E(\boldsymbol{K}) = \bar{h}^2K^2/2m^*$ is the electronic energy spectrum for an ideal 3DEG, $r_0 = eE_0/(m^*\omega^2)$ with E_0 being the strength of the radiation electric field, $\gamma = (eE_0)^2/(8m^*\bar{h}\omega^3)$ and $2\gamma\bar{h}\omega$ is the energy of the radiation field. I have used the relation $\boldsymbol{E}(t) = \partial\boldsymbol{A}(t)/\partial t = (E_0\cos(\omega t), 0, 0)$ with $E_0 = \omega A_0$. Furthermore, in the presence of the EM radiation, due to the dynamical Franz–Keldysh effect [11], the energy of the electronic system becomes $\mathcal{E} = \bar{h}^2K^2/2m^* + 2\gamma\bar{h}\omega$, shifted by $2\gamma\bar{h}\omega$, the energy of the radiation field.

Equation (1) indicates that in the presence of the EM radiation field, the electron wavefunction in a 3DEG can be separated into the space-dependent part and the time-dependent part, which implies that $|\boldsymbol{K}\rangle$ is a dynamical state and is electronically analogous to an *eigenstate* where $\boldsymbol{K}$ is the quantum number. In the $(\boldsymbol{K}, t)$ representation, the unitary operator in the absence of a scattering potential can be defined through $|\boldsymbol{K};t\rangle = U_0(t,t')|\boldsymbol{K};t'\rangle$ with the initial condition $U_0(t',t') = 1$. Thus, in the presence of the EM radiation field, the unitary operator $U_0(t,t')$ can be written as

$$U_0(t,t') = \mathrm{e}^{-\mathrm{i}[E(\boldsymbol{K})+2\gamma\bar{h}\omega](t-t')/\bar{h}}\mathrm{e}^{-\mathrm{i}r_0k_x[\cos(\omega t)-\cos(\omega t')]}\mathrm{e}^{\mathrm{i}\gamma[\sin(2\omega t)-\sin(2\omega t')]} \tag{2}$$

which describes the evolution over time of a dynamical state $|\boldsymbol{K}\rangle$ of the electronic system in the Schrödinger representation. Like in the case in the absence of the radiation field [12], the unitary operator obtained here has the following features:

$$U_0^*(t,t') = U_0(t',t) = U_0^{-1}(t,t') \tag{3a}$$

$$U_0(t,t')U_0^*(t,t') = U_0^*(t,t')U_0(t,t') = 1 \tag{3b}$$

and the composition law

$$U_0(t,t'')U_0(t'',t') = U_0(t,t'). \tag{3c}$$

Furthermore, the unitary operator satisfies

$$\mathrm{i}\bar{h}\frac{\partial U_0(t,t')}{\partial t} = H_0(t)U_0(t,t') \tag{4}$$

where $H_0(t) = [\bar{h}\boldsymbol{K} - e\boldsymbol{A}(t)]^2/2m^*$ in the $(\boldsymbol{K}, t)$ representation.

In the presence of the time-dependent scattering potential $V(t)$, the unitary operator $U(t,t')$ should satisfy

$$\mathrm{i}\bar{h}\frac{\partial U(t,t')}{\partial t} = H(t)U(t,t') \tag{5}$$

in $(\boldsymbol{K}, t)$ space. Since $U_0(t,t')$ is known, $U(t,t')$ can be determined from the operator

$$U_I(t,t') = U_0^*(t,t')U(t,t') \tag{6}$$

which is the evolution operator for the dynamical states in the interaction representation and satisfies

$$\mathrm{i}\bar{h}\frac{\partial U_I(t,t')}{\partial t} = V_I(t)U_I(t,t') \tag{7a}$$

with the initial condition $U_I(t',t') = 1$, and

$$U_I(t,t') = 1 + (\mathrm{i}\bar{h})^{-1}\int_{t'}^{t}\mathrm{d}\tau\ V_I(\tau)U_I(\tau,t'). \tag{7b}$$

Here,

$$V_I(t) = U_0^*(t,t')V(t)U_0(t,t'). \tag{7c}$$

Like in the case in the absence of the scattering mechanism, $U_I(t,t')$ obtained here has all of the properties of an evolution operator.

The integral equation given by equation (7*b*) can be solved by using the iteration approach, which reads as follows:

$$U_I(t,t') = \sum_{n=0}^{\infty} U_I^{(n)}(t,t') \tag{8a}$$

where $U_I^{(0)}(t,t') = 1$ and

$$U_I^{(n)}(t,t') = (\mathrm{i}\bar{h})^{-n} \int_{t>\tau_n>\tau_{n-1}>\cdots>\tau_1>t'} \mathrm{d}\tau_n \ \mathrm{d}\tau_{n-1} \ \cdots \ \mathrm{d}\tau_1 \ V_I(\tau_n)V(\tau_{n-1})\cdots V_I(\tau_1). \tag{8b}$$

Introducing equation (6) and equation (3*a*) into equation (8), the unitary operator for an electron gas under the EM radiations is obtained, in the presence of a scattering potential, from

$$U(t,t') = \sum_{n=0}^{\infty} U^{(n)}(t,t') \tag{9a}$$

where $U^{(0)}(t,t') = U_0(t,t')$ and

$$\begin{aligned} U^{(n)}(t,t') = (\mathrm{i}\bar{h})^{-n} \int_{t>\tau_n>\tau_{n-1}>\cdots>\tau_1>t'} & \mathrm{d}\tau_n \ \mathrm{d}\tau_{n-1} \ \cdots \ \mathrm{d}\tau_1 \ U_0(t,\tau_n)V(\tau_n) \\ & \times U_0(\tau_n,\tau_{n-1})V(\tau_{n-1})\cdots U_0(\tau_2,\tau_1)V(\tau_1)U_0(\tau_1,t'). \end{aligned} \tag{9b}$$

The expansions given by equation (8) and equation (9) are power series in $V(t)$. They converge more rapidly the closer $U(t,t)$ is to $U_0(t,t')$ (or the smaller $|V(t)|$ is). The theoretical approach used here is a generalization of the time-dependent perturbation theory [12] to the case in the presence of a time-dependent driving field such as an EM radiation. $U^{(0)}$ represents the zero-order approximation; $U^{(1)}$, $U^{(2)}$, ..., $U^{(n)}$ are, respectively, the corrections of order $1, 2, \ldots, n$ to that approximation due to the presence of the scattering potential.

3. The transition probability

With the unitary operator given by equation (9), we can calculate the probability amplitude induced by the presence of the scattering potential $V(t)$ in an electron gas subjected to an EM radiation field. Noting that $U_0(\tau_n,\tau_{n-1})$ describes the time evolution of a dynamical state $|\boldsymbol{K}_n\rangle$ of the electronic system, the probability amplitude for an electronic transition from an initial state $|\boldsymbol{K}\rangle$ to a final state $|\boldsymbol{K}'\rangle$ can be calculated via

$$\langle \boldsymbol{K}'|U(t,t')|\boldsymbol{K}\rangle = \sum_{n=0}^{\infty} \langle \boldsymbol{K}'|U^{(n)}(t,t')|\boldsymbol{K}\rangle. \tag{10}$$

After assuming that the perturbation Hamiltonian is given by $V(t) = H_j \mathrm{e}^{\mathrm{i}\omega_j t}$, where H_j is time independent and ω_j is the characteristic frequency for the jth scattering potential, the contributions, in successive orders, to the probability amplitude in the $\{H_0(t)\}$ representation are obtained as

$$\langle \boldsymbol{K}'|U^{(0)}(t,t')|\boldsymbol{K}\rangle = R_{\boldsymbol{K}'\boldsymbol{K}}(t',t)\delta_{\boldsymbol{K}',K} \tag{11a}$$

$$\langle \boldsymbol{K}'|U^{(1)}(t,t')|\boldsymbol{K}\rangle = R_{\boldsymbol{K}'\boldsymbol{K}}(t',t)\frac{V_{\boldsymbol{K}'\boldsymbol{K}}}{\mathrm{i}\bar{h}} \int_{t'}^{t} \mathrm{d}\tau_1 \ S_{\boldsymbol{K}'\boldsymbol{K}}(\tau_1) \tag{11b}$$

$$\langle \boldsymbol{K}'|U^{(2)}(t,t')|\boldsymbol{K}\rangle = R_{\boldsymbol{K}'\boldsymbol{K}}(t',t)\sum_{\boldsymbol{K}_1}\frac{V_{\boldsymbol{K}'\boldsymbol{K}_1}V_{\boldsymbol{K}_1\boldsymbol{K}}}{(\mathrm{i}\bar{h})^2}\int_{t'}^{t}\mathrm{d}\tau_1\int_{t'}^{\tau_1}\mathrm{d}\tau_2\ S_{\boldsymbol{K}'\boldsymbol{K}_1}(\tau_1)S_{\boldsymbol{K}_1\boldsymbol{K}}(\tau_2) \tag{11c}$$

and

$$\langle \boldsymbol{K}'|U^{(n)}(t,t')|\boldsymbol{K}\rangle = R_{\boldsymbol{K}'\boldsymbol{K}}(t',t)\sum_{\boldsymbol{K}_1,\boldsymbol{K}_2,\ldots,\boldsymbol{K}_{n-1}}\frac{V_{\boldsymbol{K}'\boldsymbol{K}_1}V_{\boldsymbol{K}_1\boldsymbol{K}_2}\cdots V_{\boldsymbol{K}_{n-1}\boldsymbol{K}}}{(\mathrm{i}\bar{h})^n}$$
$$\times\int_{t'}^{t}\mathrm{d}\tau_1\int_{t'}^{\tau_1}\mathrm{d}\tau_2\ \cdots\ \int_{t'}^{\tau_{n-1}}\mathrm{d}\tau_n\ S_{\boldsymbol{K}'\boldsymbol{K}_1}(\tau_1)S_{\boldsymbol{K}_1\boldsymbol{K}_2}(\tau_2)\cdots S_{\boldsymbol{K}_{n-1}\boldsymbol{K}}(\tau_n). \tag{11d}$$

Here,

$$V_{\boldsymbol{K}'\boldsymbol{K}} = \langle \boldsymbol{K}'|H_j|\boldsymbol{K}\rangle \tag{12a}$$

and

$$R_{\boldsymbol{K}'\boldsymbol{K}}(t',t) = U_0(t,t')\mathrm{e}^{\mathrm{i}[E(\boldsymbol{K}')-E(\boldsymbol{K})]t'/\bar{h}}\mathrm{e}^{\mathrm{i}r_0(k_x{}'-k_x)\cos(\omega t')} \tag{12b}$$

where $U_0(t,t')$ is given by equation (2). Moreover,

$$S_{\boldsymbol{K}'\boldsymbol{K}}(t) = \mathrm{e}^{-\mathrm{i}[E(\boldsymbol{K}')-E(\boldsymbol{K})-\bar{h}\omega_j]t/\bar{h}}\mathrm{e}^{-\mathrm{i}r_0(k_x{}'-k_x)\cos(\omega t)}$$
$$= \sum_{m=-\infty}^{\infty}\mathrm{i}^{-m}J_m(r_0(k_x{}'-k_x))\mathrm{e}^{-\mathrm{i}[E(\boldsymbol{K}')-E(\boldsymbol{K})-m\bar{h}\omega-\bar{h}\omega_j]t/\bar{h}} \tag{12c}$$

where the identity $\mathrm{e}^{\mathrm{i}z\cos x} = \sum_{m=-\infty}^{\infty}\mathrm{i}^m J_m(z)\mathrm{e}^{\mathrm{i}mx}$, with $J_m(x)$ being a Bessel function, has been used.

From equation (11*a*), we find $|\langle \boldsymbol{K}'|U^{(0)}(t,t')|\boldsymbol{K}\rangle| = \delta_{\boldsymbol{K}',\boldsymbol{K}}$, which implies that the zero-order term does not contribute to the transition probability. Introducing equation (12*c*) into equation (11*b*), the first-order contribution to the square of the probability amplitude becomes

$$R_1(\tau,t') = |\langle \boldsymbol{K}'|U^{(1)}(t,t')|\boldsymbol{K}\rangle|^2 = \frac{|V_{\boldsymbol{K}'\boldsymbol{K}}|^2}{\bar{h}^2}\sum_{m',m}\frac{J_m(r_0(k_x{}'-k_x))J_{m'}(r_0(k_x{}'-k_x))}{\Omega_m(\boldsymbol{K}',\boldsymbol{K})\Omega_{m'}(\boldsymbol{K}',\boldsymbol{K})}$$
$$\times\ \mathrm{i}^{m'-m}\mathrm{e}^{\mathrm{i}(m-m')\omega t'}[1-\mathrm{e}^{\mathrm{i}\Omega_{m'}(\boldsymbol{K}',\boldsymbol{K})\tau}-\mathrm{e}^{-\mathrm{i}\Omega_m(\boldsymbol{K}',\boldsymbol{K})\tau}+\mathrm{e}^{\mathrm{i}(m-m')\omega\tau}]. \tag{13}$$

Here $\tau = t-t'$ and $\Omega_m(\boldsymbol{K}',\boldsymbol{K}) = [E(\boldsymbol{K}')-E(\boldsymbol{K})-m\bar{h}\omega-\bar{h}\omega_j]/\bar{h}$. Hence, by definition, the first-order contribution to the steady-state (i.e., $\tau\to+\infty$) transition rate for the scattering of an electron from a state $|\boldsymbol{K}\rangle$ to a state $|\boldsymbol{K}'\rangle$ is obtained as

$$W(\boldsymbol{K}',\boldsymbol{K}) = \lim_{\tau\to+\infty}\frac{\partial R_1(\tau,t')}{\partial\tau}$$
$$= \frac{2\pi}{\bar{h}}|\langle \boldsymbol{K}'|H_j|\boldsymbol{K}\rangle|^2\sum_{m=-\infty}^{\infty}J_m^2(r_0(k'_x-k_x))\delta[E(\boldsymbol{K}')-E(\boldsymbol{K})-m\bar{h}\omega-\bar{h}\omega_j]. \tag{14}$$

For the case in which $E_0\to 0$ (and so $r_0\to 0$), because $\lim_{x\to 0}J_m^2(x) = \delta_{m,0}$, we have

$$\lim_{E_0\to 0}W(\boldsymbol{K}',\boldsymbol{K}) = \frac{2\pi}{\bar{h}}|\langle \boldsymbol{K}'|H_j|\boldsymbol{K}\rangle|^2\delta[E(\boldsymbol{K}')-E(\boldsymbol{K})-\bar{h}\omega_j] \tag{15}$$

which is the well-known result obtained in the absence of the EM field by using Fermi's golden rule.

Normally, the analysis according to the time 'centre of mass' (i.e., $\mathcal{T} = (t + t')/2$) and relative coordinates (i.e., $\tau = t - t'$) can be employed to study time-dependent electronic properties in an electron gas system. In a steady state (i.e., for $\tau \to +\infty$), the first-order transition rate obtained as equation (14) corresponds essentially to the fast approximation [13]. For the case of an EM radiation field which is periodic in time, the use of the generator $e^{ix\cos y}$ in the Bessel functions results in the complete spectrum of the square of the probability amplitude (see equation (13)) being present in the τ-direction, and in only the zeroth term of $R_1(\tau, t')$ existing in the $\mathcal{T}$-direction (here $\mathcal{T} = t'$). Consequently, when only the first-order contribution is taken into consideration, the steady-state transition rate is independent of $\mathcal{T}$ and t'.

It should be pointed out that the derivation shown above has followed the standard approaches of the time-dependent perturbation theory [12]. For the case of an electron gas driven by an EM field, a result identical to equation (14) can be obtained simply from a popularly used phenomenological approach (see the appendix).

For electron–phonon interactions in an electron gas device, we may assume that the system under study can be separated into the electron of interest and the rest of the crystal, i.e., $|\boldsymbol{K}, c; t\rangle = |\boldsymbol{K}; t\rangle|c\rangle$ where $|c\rangle$ represents the state of the crystal system. Moreover, the electron–phonon interaction Hamiltonian can be taken in the form

$$H_j = V_{\boldsymbol{Q}}(a_{\boldsymbol{Q}} e^{i\boldsymbol{Q}\cdot\boldsymbol{R}} + a^{\dagger}_{\boldsymbol{Q}} e^{-i\boldsymbol{Q}\cdot\boldsymbol{R}})$$

where $\boldsymbol{Q} = (q_x, q_y, q_z)$ is the phonon wavevector, $(a^{\dagger}_{\boldsymbol{Q}}, a_{\boldsymbol{Q}})$ are the canonical conjugate coordinates of the phonon system and $V_{\boldsymbol{Q}}$ is the electron–phonon interaction coefficient. After changing $\omega_j \to \pm\omega_Q$, where the sign $+$ $(-)$ refers to absorption (emission) of a phonon with an energy $\bar{h}\omega_Q$, the first-order steady-state transition rate for an electron–photon–phonon system in a 3DEG device is obtained as

$$W^{\pm}(\boldsymbol{K}', \boldsymbol{K}) = \frac{2\pi}{\bar{h}}\begin{bmatrix} N_Q \\ N_Q + 1 \end{bmatrix}|V_Q|^2 \delta_{\boldsymbol{K}', \boldsymbol{K}+\boldsymbol{Q}} \sum_{m=-\infty}^{\infty} J_m^2(r_0 q_x) \times \delta[E(\boldsymbol{K}') - E(\boldsymbol{K}) - m\bar{h}\omega \mp \bar{h}\omega_Q] \tag{16}$$

where $N_Q = (e^{\bar{h}\omega_Q/k_B T} - 1)^{-1}$ is the phonon occupation number.

When an electron gas is subjected to intense EM radiations, the electrons in the system can interact with the radiation field via the channels of photon emission and absorption. However, in the absence of the electronic scattering mechanism, the effect of direct optical emission and absorption by electrons is very weak, due to the strictness of the selection rule for these optical processes. When interactions between electrons and phonons are present, the absorption and emission of photons can be mediated by electron–phonon scattering events. Therefore, in an ideal 3DEG, photon emission and absorption by electrons are indirect optical processes. In equation (16), the index m corresponds to the process of m-photon absorption (emission) when $m > 0$ ($m < 0$), which implies that the electron–photon–phonon interactions in a 3DEG may lead to emission and absorption of photons via multiphoton channels. Equation (16) thus exhibits features specific to electron–photon–phonon interactions in a 3DEG structure. I highlight the point that, in sharp contrast to the isotropic transition rate obtained from using Fermi's golden rule in the absence of the EM field, the presence of the linearly polarized EM radiation results in an anisotropic electronic transition rate, characterized by the dependence of $r_0 q_x$ via a term $J_m^2(r_0 q_x)$. The physical reason behind the anisotropy of the electronic transition rate obtained here can be understood by considering the fact that in the presence of a linearly polarized radiation, the EM field polarized along a certain direction can break the symmetry of the sample geometry.

For the case of high-frequency and/or low-intensity radiations, such that $r_0 q_x \ll 1$, equation (16) can be written in the following form:

$$W^{\pm}(\boldsymbol{K}', \boldsymbol{K}) \simeq \frac{2\pi}{\hbar}\begin{bmatrix} N_Q \\ N_Q + 1 \end{bmatrix} |V_Q|^2 \sum_{m=-\infty}^{\infty} \frac{\delta_{\boldsymbol{K}', \boldsymbol{K}+\boldsymbol{Q}}}{(|m|!)^2}\left(\frac{r_0 q_x}{2}\right)^{2|m|} \times \delta[E(\boldsymbol{K}') - E(\boldsymbol{K}) - m\hbar\omega \mp \hbar\omega_Q]. \tag{17}$$

In the past, the interactions between electrons and phonons and also those between electrons and photons were treated using separate interaction Hamiltonians [8, 9]. Using this approach, the net effect of electron–phonon interactions in the presence of multiphoton processes can be calculated using the second- or higher-order perturbation theory. The rate of transitions induced by one-photon absorption process (i.e., by $m = 1$ alone) given by equation (17) is identical to that obtained from using the second-order perturbation theory [9]. This implies that the electron–photon interactions can only be treated as a perturbation when the condition $r_0 q_x \ll 1$ is satisfied. From these results, one can see that the theoretical approach used in this study is much more powerful in dealing with such problems as multiphoton processes and, therefore, is much better than the previous treatment for electron–photon–phonon interactions. For the case where the time-dependent Schrödinger equation, in which the EM field is included, can be solved analytically, it is unnecessary to treat electron–phonon and electron–photon interactions separately.

4. The electron–photon–phonon scattering rate

For polar semiconductors such as GaAs under intense THz laser irradiations, the electron interaction with longitudinal optical (LO) phonons is the principal channel for relaxation of optically excited electrons in the system. This can be seen from the facts that: (1) the LO-phonon energy in GaAs is comparable to the energy of THz photons; (2) the rate of electron–LO-phonon scattering can be on the same scale as the THz frequency [7]; (3) strong LO-phonon modes are present in GaAs; (4) the electron–LO-phonon interactions can result in an inelastic scattering and in a substantial energy transfer during electronic transitions. Furthermore, for a GaAs-based electron gas system, the frequency of the acoustic phonons associated with the deformation potential and piezoelectric oscillation modes is typically about $\omega_Q/2\pi \sim 0.1$ THz [14]. Hence one would expect the electron interactions with acoustic phonons to play an important role in relatively low-frequency and low-intensity radiations. From now on, I limit myself to consideration of the situation of electron–photon–phonon interactions via LO-phonon coupling.

4.1. Analytical results

In a polar semiconductor, the electron–LO-phonon interactions can be described by the Fröhlich Hamiltonian, from which the coupling coefficient is given by

$$|V_Q|^2 = 4\pi\alpha L_0(\hbar\omega_{\mathrm{LO}})^2/Q^2. \tag{18}$$

Here α is the electron–LO-phonon coupling constant, ω_{LO} is the LO-phonon frequency in the long-wavelength limit and $L_0 = (\hbar/2m^*\omega_{\mathrm{LO}})^{1/2}$ is the polaron radius. Furthermore, for LO-phonon oscillation modes, $\omega_Q \simeq \omega_{\mathrm{LO}}$ in the long-wavelength range. Thus, the steady-state electronic transition rate, due to the first-order contribution of electron–photon–LO-phonon interactions, becomes

$$W^{\pm}(\boldsymbol{K}',\boldsymbol{K}) = 8\pi^2\alpha L_0\bar{h}\omega_{\mathrm{LO}}^2\begin{bmatrix} N_0 \\ N_0+1\end{bmatrix}\delta_{\boldsymbol{K}',\boldsymbol{K}+\boldsymbol{Q}}\sum_{m=-\infty}^{\infty}\frac{J_m^2(r_0q_x)}{Q^2} \times \delta[E(\boldsymbol{K}')-E(\boldsymbol{K})-m\bar{h}\omega\mp\bar{h}\omega_{\mathrm{LO}}] \tag{19}$$

where $N_0=(\mathrm{e}^{\bar{h}\omega_{\mathrm{LO}}/k_BT}-1)^{-1}$ is the LO-phonon occupation number. The rate for scattering of an electron in a state $|\boldsymbol{K}\rangle$, due to the first-order electron–photon–LO-phonon interaction, is given by

$$\lambda^{\pm}(\boldsymbol{K}) = \sum_{\boldsymbol{K}'}W(\boldsymbol{K}',\boldsymbol{K}) = \sum_{m=-\infty}^{\infty}\Theta[E_m(K)]\lambda_m^{\pm}(\boldsymbol{K}) \tag{20}$$

where the contribution due to the m-photon process is

$$\lambda_m^{\pm}(\boldsymbol{K}) = \alpha\omega_{\mathrm{LO}}\begin{bmatrix} N_0 \\ N_0+1\end{bmatrix}\int_{k_xL_0-E_m^{1/2}(K)}^{k_xL_0+E_m^{1/2}(K)}\frac{\mathrm{d}x\ J_m^2(\alpha_0x)}{\sqrt{4x^2(KL_0)^2+4x(k_xL_0)(m\tilde{\omega}\pm1)+(m\tilde{\omega}\pm1)^2}}. \tag{21}$$

Here, $\Theta(x)$ is the unit step function, $\tilde{\omega}=\omega/\omega_{\mathrm{LO}}$, $E_m(K)=(KL_0)^2+m\tilde{\omega}\pm1$ and $\alpha_0=2E_0/(F_0\tilde{\omega}^2)$ with $F_0=\omega_{\mathrm{LO}}(2m^*\bar{h}\omega_{\mathrm{LO}})^{1/2}/e$ being the polaron electric field.

From equation (20), we see that when the condition $\bar{h}^2K^2/2m^*+m\bar{h}\omega\pm\bar{h}\omega_{\mathrm{LO}}\geqslant0$ is satisfied, the channel for electron–LO-phonon scattering via an m-photon process opens up. In this case, the electron–phonon interaction via absorption and emission of phonons by electrons can be accompanied by emission ($m<0$) and absorption ($m>0$) of photons. In the presence of linearly polarized EM fields, due to the anisotropic transition probability, the electron–photon–LO-phonon scattering rate is anisotropic and depends on k_x, the electron wavevector (or momentum) along the direction in which the radiation field is polarized.

4.2. The electro-photon–phonon resonance effect

For an electron gas system, the initial state $\boldsymbol{K}=0$ is the electronic state most likely to be occupied. The scattering rate for electron–photon–LO-phonon interactions in the $\boldsymbol{K}\to0$ limit is given by

$$\lambda^{\pm}(0) = 2\alpha\omega_{\mathrm{LO}}\begin{bmatrix} N_0 \\ N_0+1\end{bmatrix}\sum_{m=-\infty}^{\infty}\frac{\Theta(m\tilde{\omega}\pm1)}{m\tilde{\omega}\pm1}\int_0^{\sqrt{m\tilde{\omega}\pm1}}\mathrm{d}x\ J_m^2(\alpha_0x). \tag{22}$$

This implies that in the presence of the EM radiations, the scattering channel for electron–photon–phonon interactions without the assistance of initial electron momentum can open up when the condition $m\omega\geqslant\mp\omega_{\mathrm{LO}}$ is satisfied. This condition corresponds to the processes in which an electron gains (loses) the energy from the radiation field via absorption (emission) of photons and the electron loses (gains) the energy via emission (absorption) of phonons. When the energy that an electron can gain from absorption of photons and phonons is larger than the energy that the electron loses via emission of photons and phonons, resonance scattering for electron–photon–phonon interactions becomes possible. This type of electro-photon–phonon resonance (EPPR) is electronically analogous to the magneto-phonon resonance (MPR) [15] and electro-phonon resonance (EPR) [16] observed in the absence of an EM field and to the magneto-photon–phonon resonance (MPPR) [17] observed in the presence of radiation fields and quantizing magnetic fields.

It should be noted that in the absence of the EM radiations, LO-phonon emission scattering by an electron occurs only when $\bar{h}^2K^2/2m^*$, the electron kinetic energy, is larger than $\bar{h}\omega_{\mathrm{LO}}$, the LO-phonon energy. In contrast, for an electron gas subjected to the EM

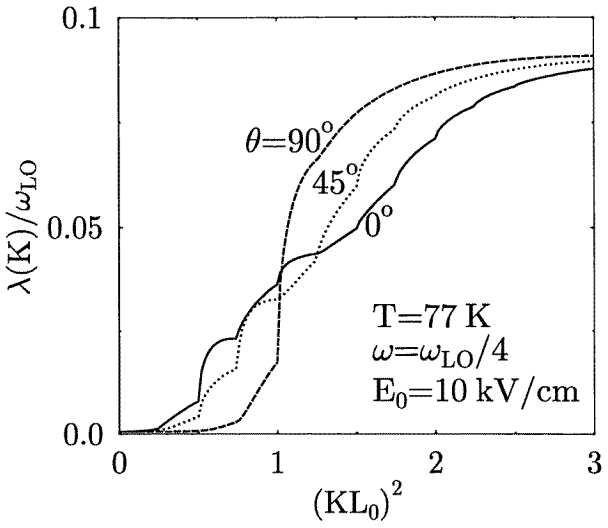

Figure 1. The total scattering rate, $\lambda(\boldsymbol{K}) = \lambda^+(\boldsymbol{K}) + \lambda^-(\boldsymbol{K})$, for electron–photon–LO-phonon interactions, as a function of the electron kinetic energy $(KL_0)^2 = (\hbar^2K^2/2m^*)/\hbar\omega_{LO}$ for different angles θ at a fixed lattice temperature T and a fixed radiation field with intensity E_0 and frequency ω. For GaAs, $\hbar\omega_{LO} = 36.6$ meV and $\omega_{LO} = 55.6$ THz. θ is the polar angle to the x-axis and $k_x = K\cos\theta$.

radiations, the LO-phonon emission scattering can be present even at $K = 0$. In this case, an electron gains the energy from the radiation field via optical absorption and the EPPR effect can be observed when $m\omega > \omega_{LO}$. For the case of LO-phonon absorption scattering, the EPPR effect can be observed via m-photon emission channels when $-m\omega < \omega_{LO}$. In the presence of the intense EM radiations, the EPPR can occur via multiphoton channels.

4.3. Numerical results

The numerical results of this paper pertain to GaAs-based 3DEG structures. The material parameters for GaAs are taken as: (1) the effective-electron-mass ratio $m^*/m_e = 0.0665$ with m_e the electron rest mass; (2) the electron–LO-phonon coupling constant $\alpha = 0.068$; (3) the LO-phonon energy $\hbar\omega_{LO} = 36.6$ meV (i.e., $\omega_{LO}/2\pi = 8.85$ THz). For GaAs, the polaron radius is $L_0 = (\hbar/2m^*\omega_{LO})^{1/2} = 39.5$ Å and the polaron electric field is $F_0 = \omega_{LO}(2m^*\hbar\omega_{LO})^{1/2}/e = 92.5$ kV cm^{-1}. In the calculations, the contributions from $m = 0, \pm1, \pm2, \ldots, \pm20$ optical processes have been taken into consideration. The inclusion of more multiphoton processes within the calculation affects only the results for low-frequency radiations.

The total scattering rate for electron–photon–LO-phonon interactions as a function of the electron kinetic energy for different angles θ is shown in figure 1 at a fixed temperature and for a fixed radiation field. Here I define $k_x = K\cos\theta$ and $(k_y^2 + k_z^2)^{1/2} = K\sin\theta$ with θ being the polar angle to the x-axis. At $T = 77$ K, the effect of LO-phonon absorption scattering is relatively weak, due to the small LO-phonon occupation number. Therefore, the scattering rate shown in figure 1 results mainly from LO-phonon emission scattering via absorption of photons by electrons in the low-energy regime and via emission of photons in the high-energy regime. In figure 1 the anisotropy for electron–photon–phonon interactions in the presence of the intense EM radiation is evident. Figure 2 shows the contribution from

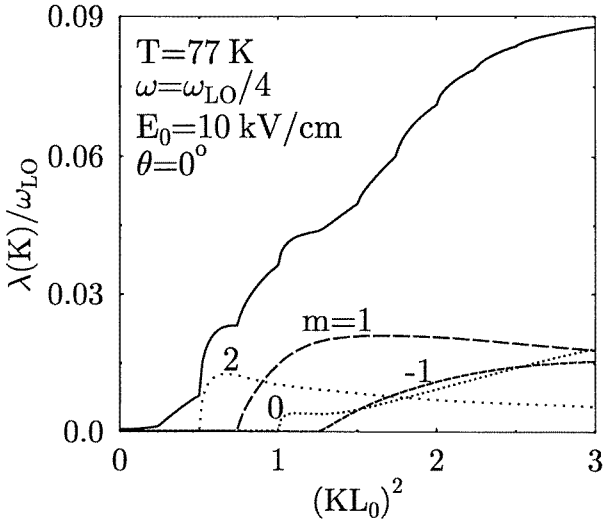

Figure 2. The contributions from different optical processes to the total electron–photon–LO-phonon scattering rate at $\theta = 0^\circ$ for a fixed radiation field. Here $m > 0$ and $m < 0$ correspond, respectively, to the processes of photon absorption and emission.

different optical processes to the total scattering rate at $\theta = 0^\circ$. At a fixed radiation field, the channel for electron–photon–LO-phonon scattering via LO-phonon emission accompanied by an m-photon process opens up when the condition $\bar{h}^2K^2/2m^* + m\bar{h}\omega > \bar{h}\omega_{LO}$ is satisfied. For the case in which $\omega = \omega_{LO}/4$, the optical process of two-photon absorption (one-photon absorption, zero-photon emission and one-photon emission) contributes to LO-phonon emission scattering when the electron kinetic energy $(KL_0)^2 = (\bar{h}^2K^2/2m^*)/\bar{h}\omega_{LO}$ is larger than 0.5 (0.75, 1 and 1.25), as can be seen in figure 2. With increasing electron kinetic energy, because more optical channels for electron–phonon interactions become accessible, the scattering rate increases. The non-monotonic increases in $\lambda(\boldsymbol{K})$ with increasing K^2, observed in figures 1 and 2 when $(KL_0)^2 \sim n/4$ with $n = 0, 1, 2, \ldots$, correspond to the opening up of channels for different optical processes.

The influences of the radiation intensity and the radiation frequency on the electron–photon–LO-phonon scattering rate at $\theta = 0^\circ$ are shown, respectively, in figures 3 and 4. At a fixed radiation frequency, the scattering rate in the low-energy regime increases with radiation intensity because of the enhancement of the strength of the photon absorption scattering. In contrast to the case in the absence of the EM radiation (i.e., the case for $E_0 = 0$ shown in figure 3, where the strong scattering is present when $\bar{h}^2K^2/2m^* > \bar{h}\omega_{LO}$), the presence of the intense EM radiations can result in a strong LO-phonon emission scattering in the energy regime where $\bar{h}^2K^2/2m^* < \bar{h}\omega_{LO}$ due to the channels for photon absorption. Within the relatively high-energy regime, the scattering rate decreases with increasing radiation intensity (see figure 3). At a fixed radiation intensity, the scattering rate in the low-energy regime increases with decreasing radiation frequency (see figure 4), because more photon absorption processes are possible for lower-frequency radiations. Within the relatively high-energy regime, the electron–photon–LO-phonon scattering rate decreases with ω. For the case of high-frequency radiations (i.e., for $\omega = \omega_{LO}$, as shown in figure 4, which looks similar to the case for $E_0 = 0$ shown in figure 3), the EM radiation field affects the electron–phonon scattering only very weakly. From equation (21), we see that

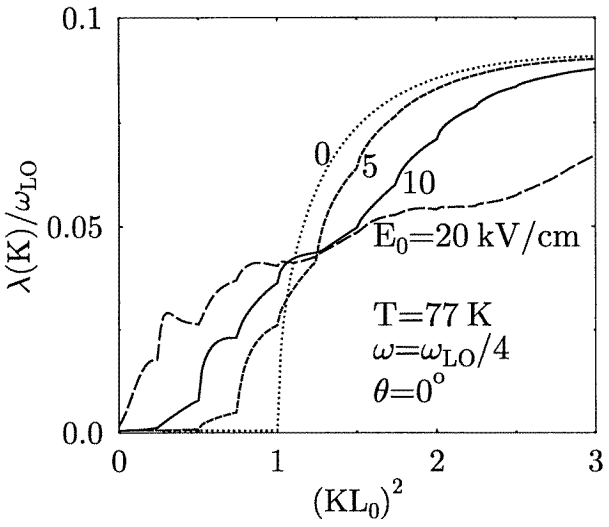

Figure 3. The influence of the radiation intensity E_0 on the total electron–photon–LO-phonon scattering rate at a fixed radiation frequency ω for $\theta = 0^\circ$. The case with $E_0 = 0$ is that in the absence of the radiation field.

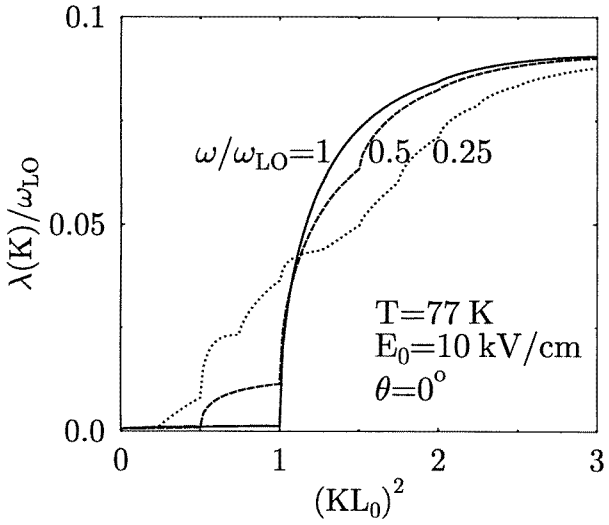

Figure 4. The influence of the radiation frequency on the total electron–photon–LO-phonon scattering rate for a fixed radiation intensity at $\theta = 0^\circ$.

the stronger modification of the electron–photon–LO-phonon interactions by the radiation field can be achieved when $\alpha_0 = 2(E_0/F_0)(\omega_{\rm LO}/\omega)^2 \sim 1$. For polar semiconductors, the polaron electronic field $F_0 \sim 100$ kV cm^{-1} is very high. Therefore, a strong effect of the radiation field on the electron–photon–LO-phonon interactions can only be observed for a relatively low-frequency radiation for a fixed radiation intensity. When $\alpha_0 \ll 1$, since $\lim_{x\to 0} J_m^2(x) = \delta_{0,m}$, only a zero-photon process contributes to the electron–photon–phonon scattering.

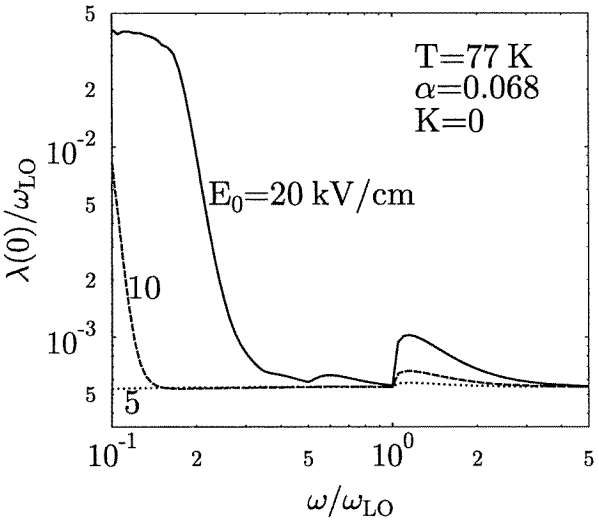

Figure 5. The total electron–photon–LO-phonon scattering rate at $K = 0$ as a function of the radiation frequency for different radiation intensities.

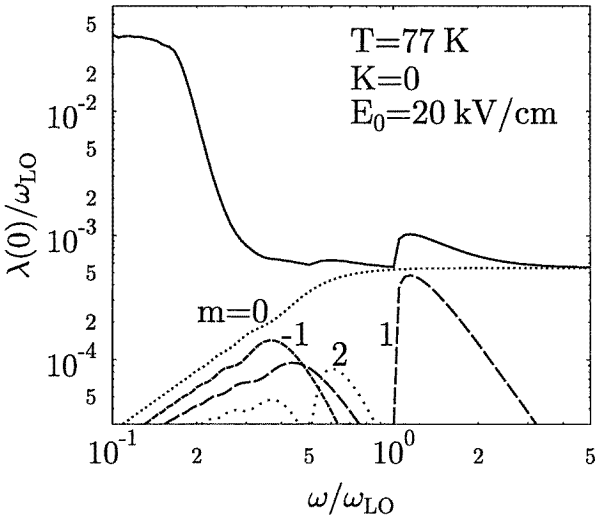

Figure 6. The contributions from different optical processes to the total electron–photon–LO-phonon scattering rate as functions of the radiation frequency for a fixed radiation intensity at $K = 0$. Here $m > 0$ and $m < 0$ correspond, respectively, to the processes of photon absorption and emission.

The electron–photon–LO-phonon scattering rate at $K = 0$ as a function of the radiation frequency for different radiation intensities is shown in figure 5. At $T = 77$ K and for $K = 0$, the electron–photon–phonon interaction within the low-radiation-frequency regime is mainly achieved via phonon emission accompanied by the absorption of photons. With increasing radiation frequency, the channels for multiphoton absorption are closing down (see figure 6) and, consequently, the total scattering rate decreases. As can be

seen in figure 6, under high-frequency irradiations the scattering is due to the processes of zero-photon and one-photon absorption. Under very high-frequency irradiations, the electron–phonon coupling is mediated by the zero-photon process alone and the scattering rate depends little on the radiation frequency (see figure 6). From figure 5, we see that for low-intensity radiations (i.e., $E_0 = 5$ kV cm^{-1}), the electron–LO-phonon interaction depends weakly on the radiation frequency.

The dependence of the electron–photon–LO-phonon scattering rate for $K = 0$ on the radiation intensity is shown in figure 7 for different radiation frequencies. With increasing radiation intensity, the total scattering rate increases because of the increase in optical absorption scattering for LO-phonon emission. In figure 8, the contributions due to different optical processes are presented. From equation (22), we see that for LO-phonon emission scattering, $\lambda^{-}(0) = 0$ when $m\omega < \omega_{\mathrm{LO}}$. Therefore, for $\omega = \omega_{\mathrm{LO}}/4$, as shown in figure 8, the process of LO-phonon emission via four-photon absorption does not give rise to electron–photon–phonon scattering for $K = 0$. The increase in the total scattering rate with the radiation intensity, shown in figure 8, results from the processes of absorption of $m > 4$ photons.

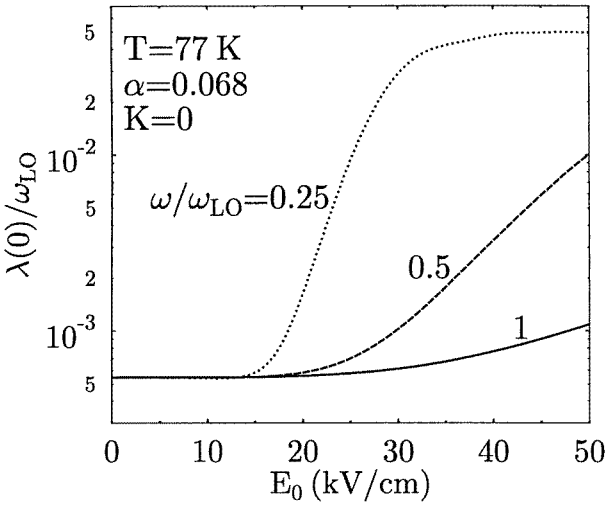

Figure 7. The total electron–photon–LO-phonon scattering rate at $K = 0$ as a function of the radiation intensity for different radiation frequencies.

The theoretical results shown in figures 5–8 indicate that the EPPR effect may be observed in a polar semiconductor under intense EM irradiations with relatively low radiation frequencies. Furthermore, the EPPR may occur via LO-phonon emission scattering accompanied by multiphoton absorption.

5. Conclusions

In this paper, I have developed a novel theoretical approach in dealing with electron–photon–phonon interactions in a 3DEG system under linearly polarized intense EM irradiations. Applying the exact solution of the time-dependent Schrödinger equation, in the absence of a scattering potential, to time-dependent perturbation theory, I have derived the unitary operator in the presence of the scattering potential. From the unitary operator obtained, the

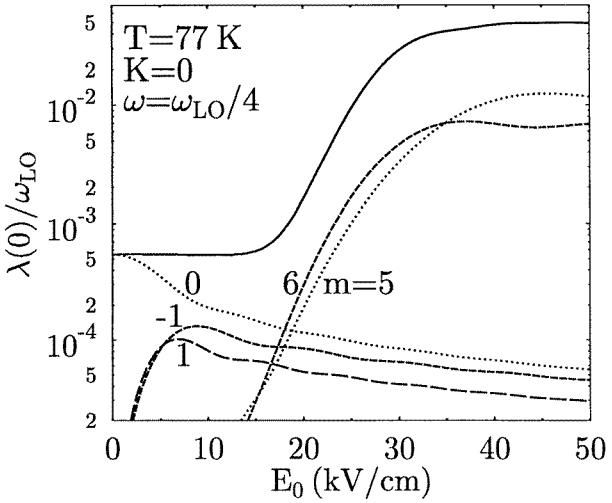

Figure 8. The contributions from different optical processes to the total electron–photon–LO-phonon scattering rate as functions of the radiation intensity for a fixed radiation frequency at $K = 0$. Here $m > 0$ and $m < 0$ correspond, respectively, to the processes of photon absorption and emission.

first-order steady-state transition probability for electron–photon–phonon interactions has been obtained. Using this theoretical approach, the effect of the EM radiation field can be considered more exactly and the effect of multiphoton processes on the interaction between electrons and photons and that between electrons and phonons in an electron gas system can be more easily and directly included. For the case of a polar semiconductor, I have studied the electron–photon–phonon interactions via LO-phonon coupling. The dependence of the electron–photon–LO-phonon scattering rate on the frequency and intensity of the EM radiation has been examined. The main results obtained from this study are summarized as follows.

For an ideal 3DEG subjected to an intense THz EM field, the electron–phonon interactions can be accompanied by the emission and absorption of photons, and these optical processes can be achieved via multiphoton channels. In this situation, the emission and absorption of photons by electrons are indirect optical mechanisms mediated by phonon scattering.

In the presence of a linearly polarized intense EM radiation, the electron–photon–phonon interaction in a 3DEG system is anisotropic and depends strongly on the electron wavevector (or momentum) along the direction in which the radiation field is polarized. From this result, one would expect the conductivity tensors in this situation to also be anisotropic.

For polar semiconductors such as GaAs, the scattering channel for electron–photon–LO-phonon interactions, via the m-photon process, opens up when the condition $\bar{h}^2K^2/2m^* + m\bar{h}\omega \pm \bar{h}\omega_{\rm LO}$ is satisfied. In sharp contrast to the case where the EM field is absent, the presence of the intense EM radiations can result in a strong LO-phonon emission scattering even in the $K \to 0$ limit due to the presence of the channels for optical absorption.

For a polar-semiconductor-based 3DEG, within the $K \to 0$ limit, the electron–LO-phonon emission (absorption) scattering can be achieved via absorption (emission) of photons by an electron. In this situation, the electro-photon–phonon resonance (EPPR) may be observed when the condition $m\omega \sim \mp\omega_{\rm LO}$ is satisfied. The numerical results obtained

indicate that, for GaAs-based structures, the EPPR effect can be measured in intense laser fields with relatively low frequencies, and the effect may be observed via multiphoton channels.

The results obtained from the present study indicate that for a GaAs-based 3DEG system under intense THz EM irradiations, the rate for electron–photon–LO-phonon scattering is of the order of 10^{12} s^{-1}, which is comparable to the radiation frequency. This implies that in a THz-driven electron gas, the electron–photon–LO-phonon interaction is the principal channel for excitation and relaxation of electrons in the system.

The strong influence of the intense THz EM radiations on electron–photon–phonon interactions in a polar semiconductor can be understood by considering the fact that the electron kinetic energy and the phonon energy in the system are comparable to the THz photon energy. As a result, conditions such as $r_0 q_x \sim 1$ and $2(\omega/\omega_{\rm LO})^2(E_0/F_0) \sim 1$ can be satisfied, so the features specific to electron–photon–phonon interactions can be observed. It should be noted that although normally the polaron electric field in a polar semiconductor is very high (e.g., for GaAs, $F_0 \sim 100$ kV cm^{-1}), the strong-polaron effect may be measured by lowering the radiation frequency to match the above conditions.

In this paper, the calculations were carried out by considering linearly polarized EM fields with the intensity $E_0 \sim 10$ kV cm^{-1} and the frequency $\omega/2\pi \sim 1$ THz. These radiation conditions can be achieved through using recently developed free-electron laser radiations. Hence, the effect of electron–photon–phonon interactions, discussed in this paper, can be observed by measuring the non-linear response of a polar semiconductor device to the frequency and intensity of the THz FEL radiations. Finally, the steady-state transition rate for electron–photon–phonon interactions, obtained from this study, can be applied to further calculations on transport and optical properties (such as the electron-energy-loss rate and conductivity) of a THz-driven electron gas by using, e.g., the Boltzmann equation approach or a Monte Carlo simulation [18]. However, this would require considerably more analytical and numerical work, which is outside the scope of this paper.

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Appendix

As suggested by one of the referees for this manuscript, the first-order transition rate given by equation (14) can also be derived by: (i) using the integral representation of the δ-function

$$\delta(x) = \lim_{t\to\infty} 2\int_0^t {\rm d}t' \, \exp(-{\rm i}xt') \tag{A1}$$

where an infinitesimal quantity iδ has been implied to make the integral converge; (ii) noting that the EM field can be included in the single-particle energies by introducing the vector potential $E(\boldsymbol{K}) \to E(\boldsymbol{K} - e\boldsymbol{A}(t)/\hbar)$; (iii) generalizing the δ-function to

$$\Delta[E(\boldsymbol{K}) - E(\boldsymbol{K}') \pm \hbar\omega_{\rm LO}] = \lim_{t\to\infty} 2\int_0^t {\rm d}t' \, \exp\left[-{\rm i}\int_0^{t'} {\rm d}t'' \, [E(\boldsymbol{K}(t'')) - E(\boldsymbol{K}'(t'')) \pm \hbar\omega_{\rm LO}]\right]. \tag{A2}$$

However, it should be noted that it is not so easy and direct to apply this simple and phenomenological approach to the calculation of the high-order contributions.

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